A CONTRIBUTION OF THE FUZZY THEORY TO FARMING SYSTEM METHODOLOGY. FUZZY VALUATION AND PERCEPTION FROM FISHERMEN IN WESTERN TOCANTINS, BRAZIL

M. ALVES DOS REYS

Department of Agricultural Economics and Social Sciences in the Tropics and Subtropics, 490C University of Hohenheim, Stuttgart, Germany and Lutheran University Centre of Palmas, CEULP-ULBRA, Palmas, Brazil.

Abstract

Farming Systems and agents in a complex adaptive system frame are examined and Fuzzy theory is presented as a tool to deal with uncertainties and ambiguity presents in the Farming System approach. Fuzzy features underlined in modelling, optimisation and in the construction of logical rules may facilitate the emergence of "best fit" solutions in terms of management of resources and environmental sustainability. An example with fishermen in Bananal Island in Brazil using MAXMIN operator and linguistic variables is presented, fish quantities were classified as normal achievability and enough quantity gathered from the ecotone.

Key words: Fuzzy theory, Farming System, Ecotone

Introduction

As a complex system, including several features, a farming system is characterised by interdependence and interactions between "agents" (individuals, public sector entities, NGO's, indigenous people, etc). Involved public sector institutions, for example, are agents that interact with each other and with other agents within society with varying levels of interdependence, crossing the boundaries of the farming systems. These interactions result non-linear effects, that is, the complex adaptive socio-economical system is greater than the sum of the interests of its constituent agents. These features could neither a priori be neglected under the cost of the descriptiveness of the model proposed nor its behaviour are not random enough to support statistical analysis.

According to Weaver (Weaver, 1968) there are three different ranges of complexity, namely organized simplicity (few variables, differential treatment), disorganized complexity (high number of variables, random entities, statistical mechanics, chaos) and the large range of systems that are considered organized complexity. An organized complex system is methodologically undeveloped in the sense that neither analytical nor statistical methods are adequate for dealing with systems that fit into it (Klir at al., 1988) and the difficulty in studying these relationships arises from the behaviors that appear when small pieces are reassembled into the larger systems they comprise. This task often proves difficult, and the interaction of the pieces, complex. In the second half of the twentieth century, scientists began to look in earnest for patterns that would help them reassemble this knowledge. The output of any treatment, however, must to be palatable to the final user, the farmer, the decision maker and according to Klir at al. (opus cited) 'since neither Newtonian nor statistical simplification strategies are applicable in the range of organized complexity, new avenues to the simplification of systems are needed. The most promising avenue, thus far, seems to rest in skilful use of the various measures of uncertainty and information that emerge from the broad framework of fuzzy set theory.'

When dealing with complexities of Farming Systems, one must to consider that the focus is on the ideal situation of the real need of human beings, looking for alternatives that contribute most to higher level of satisfaction in basic needs of poor social groups (Doppler, 2001) and researches that have been dealing with similar issues support the following questions: What are the driving forces behind this development? What are the economic, social and ecological consequences? What kind of strategies may influence this development over time? What is the limit between conservation and sustainable use? How to measure these parameters? How to manage landscape boundaries? What are the potentials of these areas? How can the needs of the individuals be brought together with the societies' requirement for ecologically stable regions?

These and subsequent questions are made up of smaller components with complex interactions and changing rules. This paradigm can be used to study events on a small scale, or on much larger scales, such as human interactions in defining culture, human interaction with climate, the evolution of ecosystems, or the integration between rural system and protected forest areas. This work intends to contribute to the descriptive analysis of the fishermen and ecotone relationship, understanding the determination of the amount of normal and enough quantity of fish gathered from the ecotone as a key step in this analysis.

This paper is divided as follows. The next sessions discuss the ecotone and farming systems relationship and the opportunity of the fuzzy approach in dealing with such a complexity; empirical results from the fuzzy approach are then presented and finally concluding remarks are made in the last session.

Ecotones and Farming Systems

Ecotones (Risser, 1995, Samways et al., 1997, Smith et al. 1997), are prototypical examples of complex adaptive systems, in which macroscopic system properties emerge from interactions among components, and may feed back to influence the subsequent development of those interactions. In this way, ecotones like Bananal Island in Brazil represent a miscellaneous of relations crossing biodiversity, sustainability and human presence, to cite only some issues. In this case, the scale of complexity requires a combination of insights both from socio-economic and from ecosystems science.

Delimiting Farming Systems is a hard work but delimit ecotone frontier (Carter, 1994) is somewhat challenging. Delimitation, however, is not the scope we are interested in, rather our point is to analyse the relations and crossing-over between these Farming Systems and ecotone. Besides, it is stated that the complex adaptive socio-economic system in the farming-ecotone is greater than the sum of the interests of its constituent agents.

From a more regional point of view, the strong interrelation between the interest of people who live in or move to such areas to make their living in using the resources available and the societies interests to protect areas from farming to keep the quality of relevant resources at a high level or improve it, is central in many zones.

The ecological components of the Farming-ecotone in western Tocantins are represented by the savannah complex, swampy and deciduous forest formations. The social component also is aggregated in Associations like rural syndicates, fishermen association, settlement association, cooperatives, etc, which, progressively, presents a product specialisation tendency. Internal organisation, market relations, and compromised cross relationship with public agents and environment seems to be a pathway to achieves sustainable outcomes. Policy networks are examined as a particular form of system, which operate within the larger complex system. Public sector institutions are treated as a significant but not the sole set of agents involved in policy networks. Contested social values, sustainability concepts, and biodiversity as a valuable outcome are identified as key factors affecting the decision makers, and also, these factors introduce uncertainty and ambiguity in interactions between agents.

Fuzzy theory and Farming Systems

The way in which these interactions are conducted is related to the level of uncertainty they represent. Where there are high levels of certainty and confidence in the outcome of decision-making, decisions are likely to be more predictable and precise, as for example, soil chemical correction, water requirements, etc.

However, due to imperfect knowledge about the social-ecological relationships cited, decision processes are more likely to conform to fuzzy sets. Fuzzy sets theory presented by Zadeh (Zadeh, 1965) describes physical variables using linguistic values and expressions. Also, instead of conventional crisp sets, Fuzzy sets permit membership scores in the interval between 0 and 1 and they are simultaneously qualitative and quantitative, calibrated to indicate degree of membership. In other words, when assessing fuzzy membership, the researcher's goal is to assert each case's degree of membership in a set, not simply to determine its position on a continuum, relative to other cases, as in variable-oriented social research. For example, instead of measuring 'household income' (a variable), a researcher might want to emphasize a degree of membership in the set of 'households that are financially secure' (Ragin, 2000). However, until now, there are few applications of fuzzy logic on social sciences (Smithson, 1988; Ragin, 2000). At same time, very few applications of fuzzy concepts to the analysis of economic problems and economic data (Gile et al., 2001), therefore it is not surprisingly to recognize that there are fewer works dealing with Farming System approach.

In terms of methodologies, fuzzy theory presents other features that are capable to deal with complex systems theory. Fuzzy logic applied in social sciences is in its infancy (Ragin, 2000), and a promissory debate one could easily see in the horizon. We argue, amongst other researches, that Fuzzy Sets are available tools in order to better understanding interactions between agents and the decision they made in the management of complex system. Specifically, understanding Farming System with its inherent complexity and inter-disciplinarily frame, Fuzzy sets could be a useful tool in aggregating data and incorporating ambiguities and uncertainties.

Into a complexity paradigm, problems concerning resources are going to be more complex in the future. Population growth, climate variability, state regulatory interference and guidance will increase the complexity on resource analysis. Foreseeing with time dynamics and interfaces between the systems enlarge even more the complexity. Besides, into an uncertainty paradigm, one must to distinguish between variability and uncertainty (Simonovic, 1997). Uncertainties caused by variability are a result of inherent fluctuations in the quantity of interest (hydrologic variables, carbon fixation, yield, land use, etc). Sources of variability are temporal, spatial and individual heterogeneity. Also, it can be difficult to separate deterministic (reducible) uncertainty from purely random (irreducible) components in spatio-temporal processes (Phillips 1999). Since natural resources present a considerable temporal and spatial variability and ecotones present a clear individual heterogeneity, one could realize the analysis of resources under the uncertainty paradigm.

Fuzzy Modeling and rules

Uncertainties also are caused by lack of knowledge. Models express selective properties of a domain of discourse for the purpose of understanding, predicting, or controlling its behavior. They are, in essence, simplify representations of real world processes and uncertainties appear from oversimplification or failure in addressing important variables of the process under analysis. Decision uncertainty arises when there is ambiguity and controversy concerning how to compare and weigh social objectives. In a temporal and spatial scale addressing

sustainability of future generations, fuzzy sets approach can successfully manage uncertainties (Simonovic, 2000).

Models are somehow neither 'realistic' nor ordinary, usually they are, in some way, artificial (Boland, 1989). In this way they are abstractions of a Universe of Discourse that ignore properties judged to be irrelevant to simplify the presentation of properties judged to be relevant. A fuzzy model captures properties of a semantic world by a syntactic representation (linguistic variables) in order to understand, predict and control its behavior. As far as incommensurables outputs still are regarded as such (security, permanency, etc), they could be considered on applying Bellman and Zadeh's concept of fuzzy environment (Bellman et al., 1970) in order to engender alternative solutions. Some works in literature also regard (or appoint to) incommensurable outputs dealt with fuzzy concepts as, for example, forestry (Mendoza et al., 1989; Hof et al., 1994), ecology (Silvert, 1997), social inequalities (Basu, 1987), and poverty (Cerioli et al., 1990).

Fuzzy logic statements could deal with interactions and rules in agents make assessments, make judgment, negotiate and, find best-fit solutions. These best-fit solutions are a central feature of outcomes in complex social and economical decision-making. An individual solution could be the best fit between a number of competing and potentially conflicting objectives, but it may not be the technically best solution according to such narrow criterion as economic efficiency. The outcomes as well may be different in each instance because different factors, agents and their judgments. For example, based on concerns arising from both environment and forestry, it was recognized that there was a need to increase research on the economic value of tropical forests to help policy makers form wise decisions on the utilization and conservation of tropical forest resources (Kramer et al., 1995), besides, in limiting factor problems, such as carbon sequestration depending on water, light and carbon dioxide (Hof et al., 1990) fuzzy aggregation operators apply very naturally and despite the fact that it does not solve the valuation problem, it can generate useful solutions that would be not generated otherwise.

Fuzzification

An expression in which linguistic variables are related to linguistic terms represents a linguistic statement (Zadeh, 1972) in Fuzzy logic. Expressions such as `Quantity is enough' or 'Market is very risky', that is, with the simple basic structure of –linguistic variable – Symbol of comparison – Linguistic term – are referred to here as linguistic statement. The determination of the matching of input variables with the linguistic terms is referred to as fuzzification (fig. 1). To this end, the actual degree of membership for input variables is determined for each linguistic term of the corresponding linguistic variable. Data from the interviewers are tabled and the process of inference starts. The inference consists of three subfunctions: aggregation, activation and accumulation. Considering no weighting factor, one could use the MaxMin inference strategy, which use the maximum for accumulation and the minimum for the algebraic product for activation. Inference process consists in aggregation and accumulation. There are, however, different procedures available in the literature.

The aggregation determines the degree of accomplishment of the condition from the degree of membership of the sub conditions. Finally, after inference process that provides a membership function as a result, one can convert this result in a crisp number using one of the techniques for defuzzification. The crisp number generated should provide a good representation of the information contained in the fuzzy set. One possibility is to determine the centre of the gravity under the membership function and the crisp output is determined as the abscissa value.



Figure 1: Inference process

Organisation of the work

This paper undergoes data collect in field survey between March and December 2000 in the North of Bananal Island, Brazil, and surrounding area. The area has specific ecological (ecotone) and human made (Farming systems) characteristics. From the data, 23 fishermen families were selected and specific questions about quantity, quality and availability of fish was analysed using fuzzy logic theory. Data from individual fishermen were aggregated using Min (AND) fuzzy operator for each question. The results were accumulated using Max (OR) fuzzy operator in a MaxMin strategy of inference. No weight between the data was considered. After accumulation, defuzzification was done based in Centre of Gravity, considering data as singleton for simplification, returning a crisp result on an enough quantity and normal achievability of fish in kilograms in a week basis.

Fishermen and Bananal Island ecotone

Fishermen as agents in Bananal Island complex system is something new, nevertheless during centuries indigenous people (also agent in the analysis) fishes in Javaes and Araguaya rivers with traditional methods. The colonization of the North of Bananal Island took also fishermen, looking for not explored areas in connection with some possible markets. Increasingly market relations in the area (Palmas city is not so far away), energy availability (new dams are now in construction), population growth, the overall economic situation in Brazil and regulations in Tocantins State are driving forces in this process. In other hand, water quality, illegal amateur fishery, chemical products on water from big farmers activities down the stream, indigenous people living in the area, settlements and subsequently pressures on the environment or - in the other hand, considered part of the ecotone as well (Pyook, 1992), and restrictions on fishery areas in Bananal Island are pressures against fishery developments, at least a sustainable one.

The formal crossing relations between the governmental authorities and the fishermen association are in their initial steps. The State looks for a solution in terms of work of these fishermen because of the plans to preserve the area as an APA (environmental protection area). Fishermen are not required to left the APA, it circumscribes also the Bananal Island Ecotone, but their development will be (at least it is supposed to) oriented and controlled. But, in the case of the Cantao Park, no extraction is foreseen. It is 89000 ha where nothing could be exploited, until now the technicians approved the area only for tourism proposes. They trying to captivate the fishermen with this alternative with the possibility they could work as guides to the tourist people.

Survey work in the area reports a population of associated fishermen of 58 families. Also, according to key persons related to the fishing activity, almost 8 in each 10 kg of fish caught come from not professional fishermen. Some alternatives arise from technicians and part of the fishermen, that is the management of the lake, which appears when the water goes down,

keeping some of them to reproduction, others as food resources, some for preservation and others to commercialisation. This and other alternatives are far from a consensus among the parts.

Fuzzy analysis of Fishermen activities

One of the aspects surveyed is the amount of fish 'enough' and at same time 'possible' to catch in the region, without specification of where, in order to 'fulfil' in a 'total extent' the requirements of his, or hers, family. Some questions arise, for example, the family component number, the quantity that the fishermen have to take from the river to buy the things and food to fulfil family requirements. Each family has each own necessities; each fisherman has each own technique or knowledge as for example, special places to fishing, in legal or illegal areas, and so on. All these instances are translated in the statements of possible and needed quantity of fish one could capture. These characteristics support the opportunity to use the fuzzy approach and because of this data collected during survey works in the area on fishermen activities are dealt with fuzzy linguistic variables.

The linguistic variable 'quantity' is described by the linguistics terms 'not enough', 'enough', 'quite enough' and 'unexpected'. Each linguistic term is described by the scale of quantity (kg of fish) (fig. 2). The same procedure for the linguistic variable 'viability ' or 'achievability ', described with linguistic terms like 'very easy', 'easy', 'normal ', 'hard' and 'very hard'. An expression in which linguistic variables are related to linguistic terms represents a linguistic statement in Fuzzy logic. Expressions such as `Quantity is enough' or 'Quantity is not enough', that is, with the simple basic structure of –linguistic variable – Symbol of comparison – Linguistic term – are referred to here as linguistic statement.



Figure 2. Description of the linguistic variable 'quantity' by linguistic terms and their hierarchy on the scale (in Kg).

In contrast to classical logic, in which statements only assume one of the Boolean states 'true' or 'false', linguistic statements in Fuzzy logic posses a degree of membership. Under regular set theory, elements either belong to some particular set or they do not. In contrast, in the fuzzy sets, the degree of membership may be any value between zero and unity, and an element may be associated with more than one set as for example, one would wish to differentiate between situations of surplus and deficiency in respect to a determined level of gathered fish per household. In traditional set theory the differentiation should be made through: $S_s = \{x: x > x^e\}$; $S_d = \{x: x < x^e\}$, $S_e = \{x^e\}$, that is the sets of above, under and in an equilibrium situation. A particular result, say 40 kg would be unquestionably in one, and only one of these sets. In a fuzzy set approach, however, the same sets S_s , S_d , and S_e do not have

sharp boundaries and the value 40 kg, to the fishermen, would be connected under some degree with each of these sets.

Further, empirical knowledge obtained from the interviews can be defined in rules in the form: IF condition A then conclusion B. In this context, the condition of each rule comprise a combination of linguistic statements like 'Quantity is not enough' and 'Achievability is very easy ' via input variables, while the conclusion determines the output variable in the sense of an instruction act. The result of the rules must be combined one with another via mathematical operators and these relationships between fuzzy sets and operations with fuzzy sets are governed by the membership functions. As stated, different fishermen have different vision about the 'enough' quantity of fish. Fig.3 and fig. 4 show the views of different fishermen about 'normal' achievability.

As an example, consider the table 1, which describes the enough quantity of fish for a determined fisherman family at Caseara city. In the table one can see that 20 kg of fish is not into the fuzzy set 'enough', and 30 kg has a membership value of 0,8 into this set. This means that the fisherman, according to the priorities and necessities he (or she) knows the family faces, do not consider 20 kg sufficient at all, and that 30 kg, with some adjustments into these priorities could be considered enough. Further, in the other extreme, 70 kg is far from a 'enough' quantity of fish, and 80 kg has a membership value of zero. These membership values mean that fishermen consider these quantities in kilograms as surpluses to the requirements they have.

Quantity	(kg)	20	30	40	50	60	70	80
Membership	m	0	0,8	1	1	0,8	0,5	0

Table 1: Membership values of enough quantity of fish, Caseara city, 200)0.
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These determinations of the matching of input variables (kg of fish) with the linguistic terms are referred to as fuzzification. To this end, the actual degree of membership (m) for input variables is determined for each linguistic term of the corresponding linguistic variable Data from the interviewers are tabled and the process of inference starts.

The inference consists of three sub-functions: aggregation, activation and accumulation. Considering no weighting factor, one could use the MaxMin inference strategy, which use the maximum for accumulation and the minimum for the algebraic product for activation. The aggregation determines the degree of accomplishment of the condition from the degree of membership of the sub conditions. In this case, an AND combination (fig. 8) of sub conditions (fig. 7) was used.

The same procedure was done for the linguist statement 'normal' achievability of fish (kg), that is, plotting the membership function for the sample aggregated by an AND fuzzy operator. Fig. 9 represents the accumulation sub function in the MaxMin inference strategy for the 'enough' quantity and 'normal' achievability in the G universe of discourse (kg of fish) through the OR operator. The union of two fuzzy sets A and B is defined as a function of the form u: $[0,1] \times [0,1] \rightarrow [0,1]$. The arguments of the function are the membership's values in the set A and B. and the operator MAX as the union operator was considered using the Yager class operator (Yager, 1980) defined as follows: $U_w(a,b) = \min[1, (a^w + b^w)^{1/w}]$ considering w= ∞ .

Finally, after inference process that provides a membership function as a result, one can convert this result in a crisp number using one of the techniques for defuzzification. The crisp number generated should provide a good representation of the information contained in the fuzzy set. One possibility is to determine the Centre of the gravity under the membership function and the crisp output is determined as the abscissa value, fig 10. For simplification the equation was generated by singletons and the result is U=53,6 kg of fish captured as a normal and enough quantity in a weekly basis.



Figure 3: Membership function of the term 'enough' quantity of fish (kg), fishermen family.



Figure 4: Membership function of the term 'enough' quantity of fish (kg) for 2 different fishermen family.



Figure 5: Membership function of the term 'normal' achievability of fish (kg).



Figure 6: Membership function of the term 'normal' achievability of fish (kg)



Figure 7: Membership function of the term 'enough' quantity of fish (kg) for the fishermen sample



Figure 8: Aggregation with AND fuzzy operator for 'enough' quantity (kg) for the fishermen sample



Figure 9: Accumulation with OR (Max) fuzzy operator for 'enough' quantity and 'normal' achievability of fish (kg) for the fishermen sample



Figure 10: Defuzzification

Conclusion

Fuzzy theory highlight the imprecision and ambiguities of socio-economics concepts asking for their representation. Fuzzy sets improve the link between theory and data analysis, calibrating and fitting theoretical knowledge into membership function using diversityoriented approach. Supporting diversity of cases (people, families, fishermen, environment, techniques, and so on), escaping from homogenizing assumptions, fuzzy theory is a tool to deal with systems complexity in general. In particular, dealing with Farming Systems and ecotones complexity issues, fuzzy features permits an enhancement in aggregating human activities in sensitive ecological areas in line with socio-economics and environmental theories. Referring to resource analysis, Fuzzy theory can offer some tools in order to address the main problems of the definition of an ideal level of satisfaction and the quantification and qualification of goods and services on one side and consumptive needs and restrictions on the other. Besides, Fuzzy theory can offer a set of tools in order to deal with the Farming Systems methodology in terms of optimisation, modelling, spatial analysis and inferences with fuzzy rules. The MaxMin inference strategy obtained a quantity of fish considered as normal and enough by the fishermen considering the circumstances they live and the knowledge they have. For this reason, the output generated (53,6 kg of fish) is substantially more reliable and useful to the researcher in the descriptive analysis and in the construction of possible future scenarios.

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